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Experimental formulation of four poles of three-dimensional cavities and its application

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Abstract

While the four-pole approach is a very convenient concept in modeling acoustic systems, its application has been limited mainly to systems composed of only one-dimensional (1-D) and lumped parameter elements due to the difficulty in formulating four poles of three-dimensional (3-D) cavities. In this work, an experimental procedure is developed to obtain four poles of a 3-D cavity from the measured pressure response functions. The procedure is validated by comparing the four poles obtained experimentally for a rigid-walled rectangular cavity with those obtained by analytical and numerical procedures. Establishing an experimental procedure for four-pole formulation is significant as it enables application of the four-pole approach to virtually any acoustic systems. The concept of hybrid modeling, which is building the system model by combining experimental, numerical and analytical models, is demonstrated through a simple example as the best application of the four-pole approach.

1. Introduction

The four-pole method presents the equation of an acoustic system by the relationship between the harmonic pressure and volume velocity pairs at the input and output ports in the frequency domain. The cascading property resulting from this setup enables not only efficient computation but also easy integration of sub-system models formulated by various different approaches. Despite these advantages, application of the approach has been largely limited to systems composed of lumped parameter and one-dimensional (1-D) acoustic elements partially due to the difficulty in formulating four poles of three-dimensional (3-D) cavities.

An experimental method is an attractive option to formulate four pole parameters because it can be applied to any general acoustic systems. To and Doige [1,2] first developed an experimental procedure to measure four pole parameters of acoustic systems based on the two-load method. Lung and Doige [3] generalized the approach to acoustic systems with the presence of a significant mean flow. The two-load method was prone to errors in the low-frequency range. Later, Munjal and Doige [4] developed an improved method that they called the two-source location method, which is similar to the method developed in this paper. In their

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method, four-pole parameters are calculated from the pressures measured at the input and output ports of the system and at two more points in the 1-D pipes attached to the system. As the four poles of the pipes are known in exact forms, the relationship between each pair of the measured pressures provides one equation without increasing the number of unknowns. These equations are solved to obtain the four poles of the system. However, the equations can become ill-conditioned especially at low frequencies resulting in the relatively large errors seen in Refs. [3,4], which is because the perturbed system obtained by attaching a pipe is not sufficiently distinct acoustically from the original system.

Kim and Soedel [5] proposed a procedure to formulate four poles of a general acoustic system in terms of pressure response functions of the blocked system. As the procedure formulates four poles in terms of pressure response functions, any analytical, numerical or experimental method that obtains pressure response functions can be used to implement the procedure. Wu and Zhang [6] implemented the procedure using the boundary element method (BEM) to formulate four poles of expansion mufflers. Later, Zhou and Kim [7] used the mode superposition method to obtain four poles of a rectangular cavity, and pointed out the difficulty in implementing an analytical procedure to 3D systems due to the singularity at the source point. They showed that a surface source model has to be used to obtain a bounded pressure response solution at the source point [7].

Kulkarni et al. [8] developed an experimental procedure. The procedure essentially duplicates the theoretical procedure proposed by Kim and Soedel [5] experimentally. In the method, pressure response functions are obtained as the ratios of the pressures to the volume velocity measured by the two-microphone method. When the method was applied to a 1-D pipe, the method provided accurate results in a broad frequency range. One drawback of the procedure is that it cannot be used for systems with a significant mean flow because it requires measurement of blocked pressure response functions. However, the simplicity and accuracy of the procedure makes it an excellent choice for systems with a relatively low mean flow, which is the case of most 3-D cavities.

In this paper experimental, analytical and numerical methods are applied to an identical 3-D cavity to compare the four poles obtained by the three methods to validate the experimental method developed by Kulkarni et al. [8] A rectangular hard-walled cavity, one of very few 3-D systems whose pressure response solutions can be obtained by all three methods, was built for this purpose. Comparing the four poles obtained by these three methods also helps to understand the limitations and merits of the three methods.

Among the three approaches, the analytical approach is the most cost-effective to implement; however is limited to only a few simple systems. The experimental approach is the most general; however the most expensive and time consuming. A numerical model provides an option between them. Formulation of the system model by optimally combining the four poles obtained by the three approaches is an attractive option to analyze complex acoustic systems. Enabling such a hybrid modeling was the motivation of this work.

2. Formulation of four poles of 3-D cavities

For the acoustic system shown in Fig. 1, the four-pole equation is defined as

$$\begin{Bmatrix} Q_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} Q_2 \\ P_2 \end{Bmatrix}, \tag{1}$$

where subscript 1 and 2 indicate the input and output ports, Q and P are the complex harmonic amplitudes of the volume flow velocity and pressure, and A, B, C, and D are called four poles [9]. Kim and Soedel showed that the four poles could be formulated from the pressure response functions as follows [5]:

$$A = \frac{f_{22}}{f_{21}}, \quad B = \frac{1}{f_{21}}, \quad C = -f_{12} + \frac{f_{11}}{f_{12}} f_{22}, \quad D = \frac{f_{11}}{f_{12}}, \tag{2}$$

where f_{ij} is the pressure induced at the port i in response to the unit volume flow input to the port j while the other port is closed. For example, f_{12} is the pressure induced at the input port location by the unit volume flow input to the output port while the input port is blocked. Eq. (2) implies that four poles of any acoustic systems can be derived if its pressure response functions are known.

A hard-walled rectangular box is one of very few 3-D systems whose pressure response functions can be formulated by all three methods, i.e., theoretical, numerical, and experimental methods. A rectangular

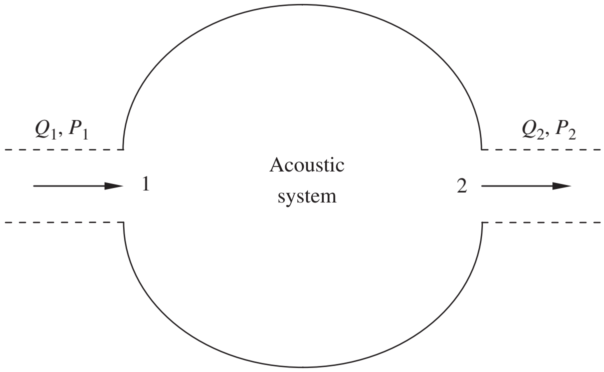

Fig. 1. Schematic representation of an acoustic system.

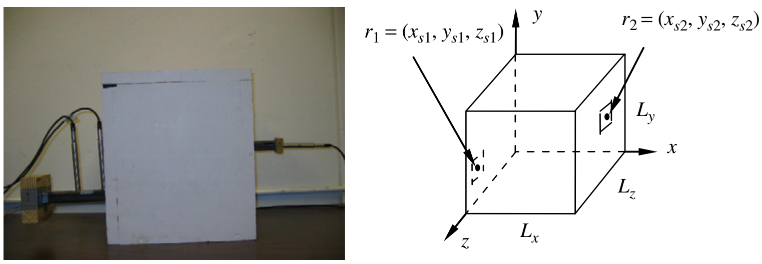

Fig. 2. The rectangular cavity used in the study.

hard-wood box shown in Fig. 2 is chosen in this study to compare the four poles formulated based on analytical, numerical and experimental pressure response functions. The system parameters are given as follows:

(1) Dimensions of the cavity: $L_x = 0.2794\,\text{m}$, $L_y = 0.3302\,\text{m}$, and $L_z = 0.2286\,\text{m}$.
(2) Acoustic medium: density $\rho_0 = 1.21\,\text{kg/m}^3$, speed of sound $c_0 = 343\,\text{m/s}$ (standard air).
(3) Input point location: $(r_1) = (x_{s1}, y_{s1}, z_{s1}) = (0, 0.0762, 0.0381)\,\text{m}$
(4) Output point location: $(r_2) = (x_{s2}, y_{s2}, z_{s2}) = (L_x, 0, 1778, 0.1524)\,\text{m}$.
(5) The cross-section of the input and output pipes, the source surface, is a square section of $0.0195\,\text{m} \times 0.0195\,\text{m}$.

2.1. Analytical formulation of four poles

The pressure response functions of the cavity can be obtained by solving the non-homogeneous wave equation [5,10] for the given mass flow source distribution $\dot{m}$,

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\partial \dot{m}}{\partial t}, \tag{3}$$

where p is the acoustic pressure and c_0 is the speed of sound. The source surface is shown in Fig. 3; therefore the mass flow can be described as follows:

$$\begin{aligned}\dot{m} = \rho_0 \frac{Q_0}{b_s c_s}\delta(x - x_s)\left[H\left(y - y_s + \frac{b_s}{2}\right) - H\left(y - y_s - \frac{b_s}{2}\right)\right] \\ \times \left[H\left(z - z_s + \frac{c_s}{2}\right) - H\left(z - z_s - \frac{c_s}{2}\right)\right]\mathrm{e}^{\mathrm{j}\omega t}.\end{aligned} \tag{4}$$

where $\delta(\cdot)$ is the Dirac delta function, $H(\cdot)$ is the unit step function and Q_0 is the total input volume flow. It is assumed that the source strength is uniform over the source surface.

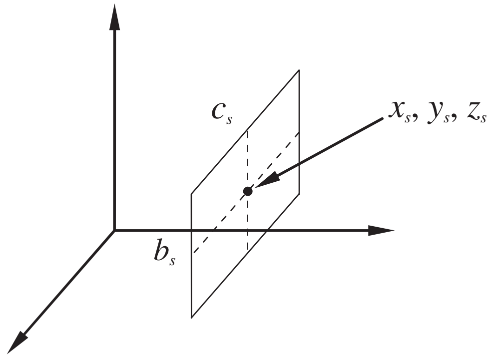

Fig. 3. Geometry of the surface source [7].

Assuming the rigid boundary condition on the surface, the pressure response solution can be obtained as follows by applying the mode superposition method to Eq. (3) [7]:

$$\frac{P(x,y,z;x_s,y_s,z_s)}{Q_0}=\frac{\mathrm{j}\rho_0 c_0^2\omega}{b_s c_s}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{P_{lmn}(x,y,z)\cos(l\pi x_s/L_s)C_m C_n}{N_{lmn}[(\omega_{lmn}^2-\omega^2)+2\mathrm{j}\omega\omega_{lmn}\xi_{lmn}]},\tag{5}$$

where *x,y,z* define the field point and x_s, y_s, z_s define the source point, and,

$$C_m=\begin{cases} b_s & \text{if} \quad m=0,\\ \dfrac{2L_y}{m\pi}\cos\dfrac{m\pi y_s}{L_y}\sin\dfrac{m\pi b_s}{2L_y} & \text{if} \quad m\neq 0,\end{cases}\tag{6}$$

$$C_n=\begin{cases} c_s & \text{if} \quad n=0,\\ \dfrac{2L_z}{n\pi}\cos\dfrac{n\pi z_s}{L_z}\sin\dfrac{n\pi c_s}{2L_z} & \text{if} \quad n\neq 0.\end{cases}\tag{7}$$

$P_{lmn}=\cos(l\pi/L_x)\ \cos(m\pi/L_y)\ \cos(n\pi/L_z)$, ω_{lmn} and ξ_{lmn} are the *lmn*th mode shape, natural frequency and modal damping ratio. $N_{lmn}=\int_V P_{lmn}(x,y,z)^2\mathrm{d}V$.

f_{11} and f_{12} are obtained by substituting $(x_s,\ y_s,\ z_s)=(x_{s1},\ y_{s1},\ z_{s1})$ to Eq. (5), i.e.:

$$f_{11}=\frac{P(x_{s1},y_{s1},z_{s1};x_{s1},y_{s1},z_{s1})}{Q_0},\quad f_{12}=\frac{P(x_{s1},y_{s1},z_{s1};x_{s2},y_{s2},z_{s2})}{Q_0}.\tag{8}$$

With $(x_s,\ y_s,\ z_s)=(x_{s2},\ y_{s2},\ z_{s2})$, f_{21} and f_{22} are obtained as follows:

$$f_{21}=\frac{P(x_{s2},y_{s2},z_{s2};x_{s1},y_{s1},z_{s1})}{Q_0},\quad f_{22}=\frac{P(x_{s2},y_{s2},z_{s2};x_{s2},y_{s2},z_{s2})}{Q_0}.\tag{9}$$

As $f_{12}=f_{21}$, only three pressure response solutions are independent.

The infinite series in Eq. (5) is approximated by a finite series by taking a sufficiently large number of terms up to the mode number *N*. Solutions converge rather quickly for f_{12} and f_{21} but very slowly for f_{11} and f_{22} in general. This difficulty is expected if it is considered that f_{11} and f_{22} are source point impedances that become infinite for the point source case [11]. The general form of the pressure response to a point source in the cavity is

$$p(r,t)=\mathrm{j}\rho_0\omega\frac{Q}{4\pi r}\mathrm{e}^{\mathrm{j}(\omega t-kr)}+\Xi(r,t),\tag{10}$$

where *r* is the distance from the source point to the filed point and $\Xi(r,\ t)$ is a function that represents the reverberation effect in the cavity. As $\Xi(r,\ t)$ is finite, it is clear that the pressure response at the source point, where $r=0$, will become infinite. Thus, convergence of the pressure solution is very slow, especially when the dimension of the source is small compared to the other dimensions of the cavity and the wavelength of interest. Fig. 4 shows f_{12} and f_{11} as functions of N, the upper limit of the modes, used in the mode superposition. For example, f_{11} in Fig. 4 corresponding to $N = 10$ is the value obtained by using 10^3 modes. As one can see, 64 terms ($N = 4$) provide satisfactory convergence for f_{12}. For f_{11}, $N = 30$ (30^3, 27,000 modes) at 100 Hz, 40 (40^3, 64,000 modes) at 500 Hz, and 120 (120^3, about 1.7 million modes) at 1000 Hz are necessary to obtain converged solutions.

How to define the pressure response function is another problem because the pressure over the source surface is not constant. Fig. 5 shows the distribution of the pressure on the source surface at the frequency of 100 Hz. The pressure response function is calculated by the average pressure value as follow [7]:

$$f_{ii} = \frac{P_{\mathrm{avg}}}{Q} = \frac{\int_S P(r)\mathrm{d}A/b_s c_s}{Q}. \tag{11}$$

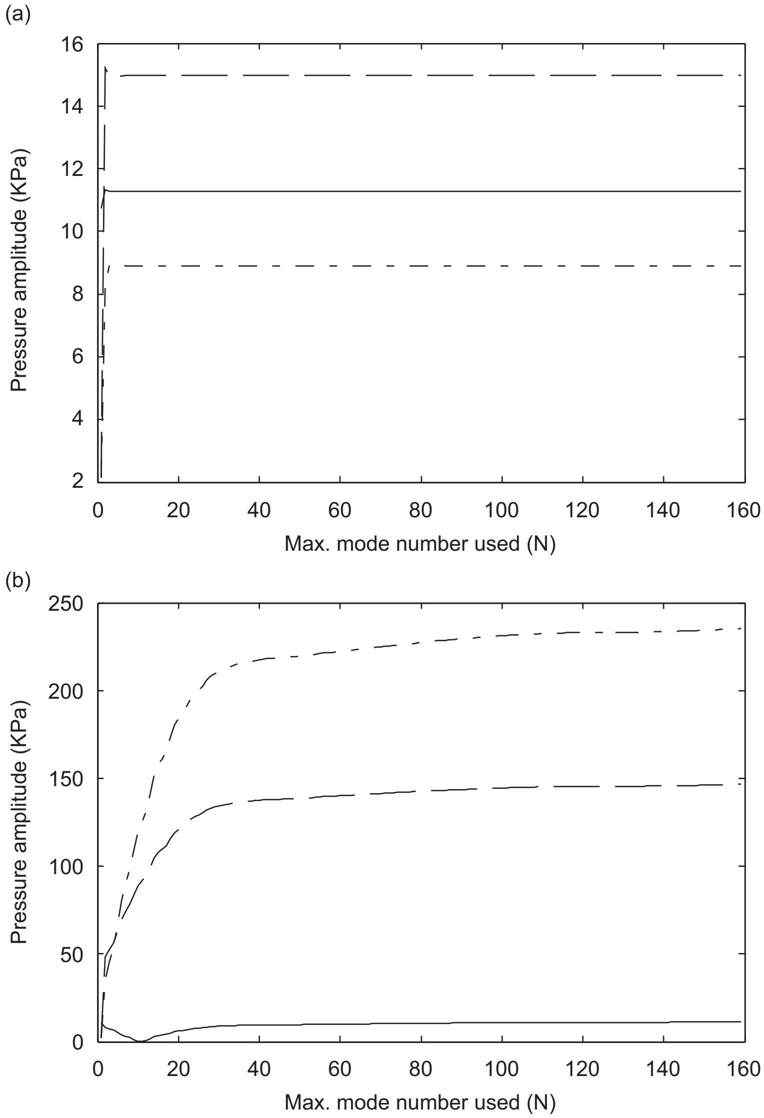

Fig. 4. Pressure response of three-dimensional cavity to surface source: (a) at far-field point, (b) at source point, ——— 100 Hz, — — — 500 Hz, —·—·-· 1000 Hz.

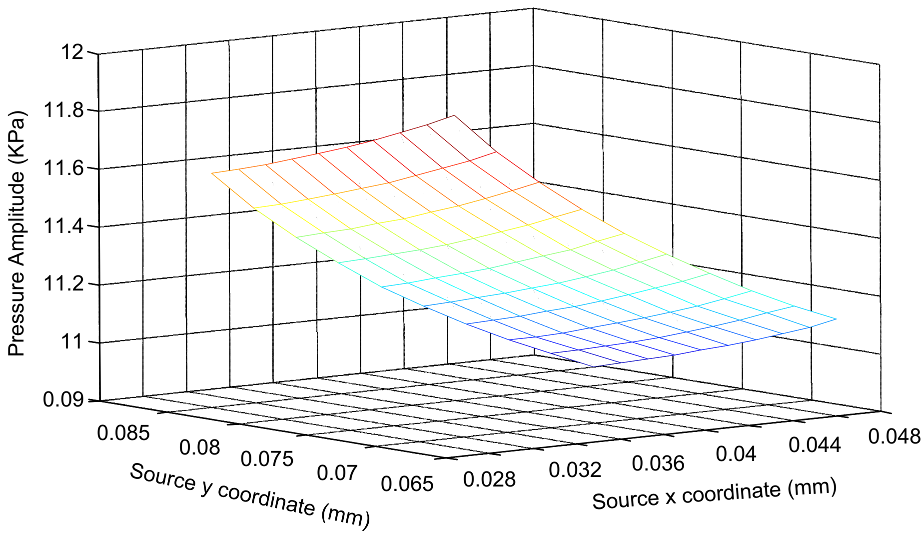

Fig. 5. Pressure distribution on the source surface.

2.2. Numerical formulation of four poles

A numerical procedure such as the finite element method (FEM) or BEM can be used to obtain the pressure response functions of 3-D cavities. In this case also, the source modeling presents a similar difficulty as in the analytical formulation, which requires to use a very fine mesh on the source surface. This problem can be avoided by attaching a short pipe each at the input and output ports as seen in Fig. 6. Because a plane wave develops in the pipe, the pressure on the source surface becomes nearly uniform.

After finding the four poles of the entire system composed of the cavity and the pipes, four poles of the cavity can be found using the cascading property of four poles. Four poles of the entire system are:

$$\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} A_{p1} & B_{p1} \\ C_{p1} & D_{p1} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_{p2} & B_{p2} \\ C_{p2} & D_{p2} \end{bmatrix}. \tag{12}$$

On the right-hand side of Eq. (12), the first and third matrices are the four poles of the pipes attached to the cavity at the input and output sides, and the second matrix is the four poles of the cavity that we are trying to find. The four poles of the 1-D pipe are known in exact forms as follows [12]:

$$\begin{bmatrix} A_{pi} & B_{pi} \\ C_{pi} & D_{pi} \end{bmatrix} = \begin{bmatrix} \cos\, kL_{pi} & \dfrac{\mathrm{j}S_i \sin\, kL_{pi}}{\rho_0 c_0} \\ \dfrac{\mathrm{j}\rho_0 c_0\, \sin\, kL_{pi}}{S_i} & \cos\, k_i L_{pi} \end{bmatrix}. \tag{13}$$

where $i = 1,2$, $k = \omega/c_0$ is the wavenumber, c_0 is the speed of sound, L_{pi} is the length of the pipe, ρ_0 is the density of the acoustic medium. After finding the four poles A_T, B_T, C_T, and D_T, of the entire system from the FEM analysis, the four poles of the cavity are easily obtained as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{p1} & B_{p1} \\ C_{p1} & D_{p1} \end{bmatrix}^{-1} \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} \begin{bmatrix} A_{p2} & B_{p2} \\ C_{p2} & D_{p2} \end{bmatrix}^{-1}. \tag{14}$$

Because the volume flow velocities in the inlet pipe and outlet pipe have to be measured in the experimental formulation, the setup shown in Fig. 6 is what is used in the experimental procedure also.

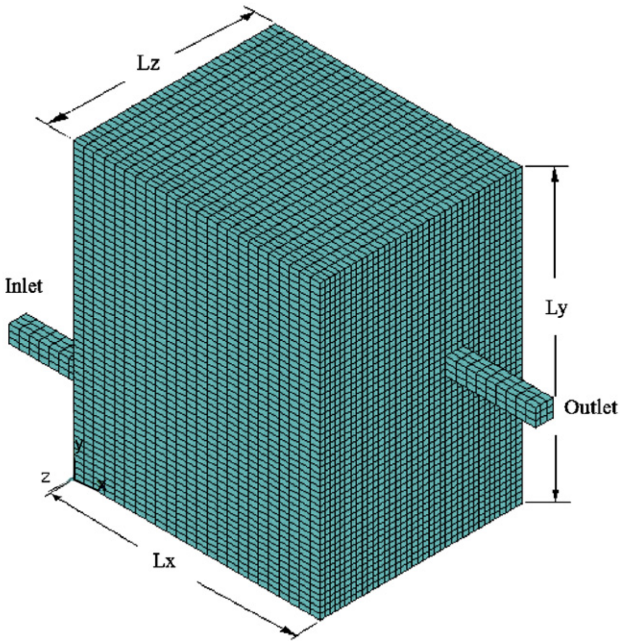

Fig. 6. Finite element model of the system.

2.3. Experimental formulation of four poles

Fig. 7 shows the three microphone setup used in this work to measure four poles. Four poles measured from this setup are the four poles of the system composed of the pipes of length L_{p1} and L_{p2} attached to the cavity. Notice that L_{p1} and L_{p2} are effective lengths of the pipes, which include the end corrections to account for the attached mass effect at the end of pipes. The procedure explained in Eq. (14) of the previous section can be applied exactly the same way to obtain the four poles of the cavity from the system four poles that will be measured. The configuration shown in Fig. 7 is to measure f_{11} and f_{21}. The pair of microphones in the driving side measures the particle velocity ($U_i = Q_i/S$) and the pressure P_i, and the microphone at the closed end side measures the pressure P_j. Two pressure response functions are obtained from this setup as follows:

$$f_{11} = \frac{P_i}{Q_i}, \quad f_{21} = \frac{P_j}{Q_i}, \tag{15}$$

where Q_i is the volume flow velocity at the input location. The other two pressure response functions, f_{22} and f_{12}, are estimated by repeating the measurement after swapping the pipes. L_{p1} and L_{p2} have to be kept the same for this swapping method possible.

The volume velocity necessary to measure the pressure response functions is calculated from the particle velocity measured by the two-microphone method [13,14]. Referring to Fig. 8, the pressure in the pipe has two traveling wave components as follows:

$$p(x,t) = (P_+ \mathrm{e}^{-\mathrm{j}kx} + P_- \mathrm{e}^{\mathrm{j}kx})\mathrm{e}^{\mathrm{j}\omega t}. \tag{16}$$

Microphones 1 and 2 measure total pressures; therefore;

$$P_1 = P_+ + P_-, \quad P_2 = P_+ \mathrm{e}^{-\mathrm{j}ks} + P_- \mathrm{e}^{\mathrm{j}ks}, \tag{17}$$

where P_1 and P_2 are the pressures measured by microphones 1 and 2 and s is the spacing between them. Rearranging Eq. (17), it can be shown that the ratio between the pressure amplitudes of the backward and

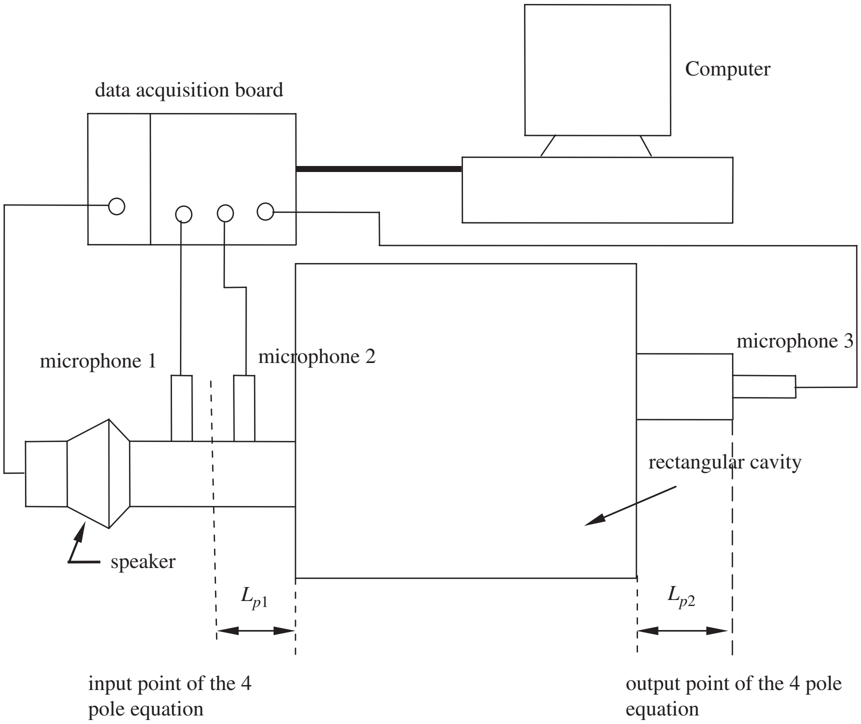

Fig. 7. Basic setup for experimental testing.

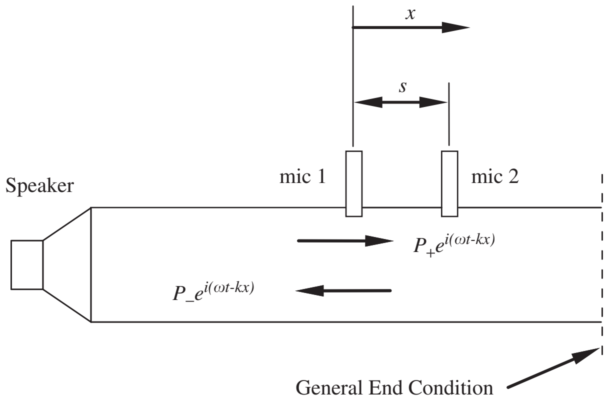

Fig. 8. Two-microphone method to measure the particle velocity.

forward traveling waves is

$$R = \frac{P_-}{P_+} = \frac{P_2/P_1 - \mathrm{e}^{-\mathrm{j}ks}}{\mathrm{e}^{\mathrm{j}ks} - P_2/P_1}. \tag{18}$$

The pressure ratio P_2/P_1 can be estimated from the transfer function between the two microphones, H_{12}, because:

$$H_{12} = \frac{P_2 P_1^*}{P_1 P_1^*} = \frac{P_2}{P_1}. \tag{19}$$

After R is estimated, the volume velocity at the location of microphone 1 can be found:

$$Q_1 = SU = S\frac{P_+ - P_-}{\rho c} = S\frac{P_+ + P_-}{\rho c}\frac{P_+ - P_-}{P_+ + P_-} = S\frac{2P_1}{\rho c}\left(\frac{1-R}{1+R}\right). \tag{20}$$

Now with Q_1 available, the pressure response function f_{21} is obtained as

$$f_{21} = \frac{P_3}{Q_1} = \frac{P_3}{P_1}\frac{P_1}{Q_1} = f_{11}H_{13}, \tag{21}$$

where H_{13} is the transfer function between the microphone 1 at the inlet and microphone 3 at the out port. As discussed by Chung and Blaser [13,14], the sensor switching technique can be used to eliminate the phase-mismatch error in estimating the cross-spectral density, that is

$$H_{12} = (H_{12}H'_{12})^{1/2}, \tag{22}$$

where prime indicates transfer functions measured after switching two microphones.

A study of the singular condition of pressure reflection coefficient by Chung and Blaser [13,14] has shown that pressure reflection coefficient in Eq. (18) becomes indeterminate as $H_{12}–e^{jks} = 0$ at the discrete frequencies at which the microphone spacing is an integer multiple of the half-wavelength of the signal. In order to avoid this condition up to the frequency f_m, the microphone spacing s must be chosen such that:

$$s \leqslant \frac{c}{2f_m}, \tag{23}$$

where c is the speed of sound and f_m is the cutoff frequency. The microphone spacing of 6.3 cm was chosen in this work which give a cutoff frequency of 2722 Hz.

3. Four poles of 3-D cavities

3.1. Comparison of four-poles obtained by three methods

Four poles of the rectangular cavity obtained by three different methods are compared in Fig. 9. In the analytical and numerical calculations, 0.5% modal damping ratio was chosen arbitrarily to account for the damping effect. The three sets of four poles show quite good agreements to one another. Much better agreement is observed between the numerical and analytical four poles, which are expected because the same modal damping ratio was used in both analyses. The large difference between the measured four poles and the analytical and numerical four poles observed in the very low-frequency range, up to 30 Hz, is considered due to the finite difference approximation employed in estimating the particle velocity. Damping effect, which was not accurately modeled, is believed to have caused the difference observed in most of the frequency range. The 770 Hz, at which the differences between the models are more pronounced, is the resonance frequency of the two pipes attached to the cavity as the effect of damping is magnified at this frequency.

While no accurate error analysis is possible, comparison of the pole A provides an estimation of the accuracy of the three procedures. We consider a hypothetical case that the cavity is driven at the input port and the output port is exposed to the open air, which is approximately described by Eq. (1) with $P_2 = 0$. Notice that this pressure release condition is physically not realizable. The condition will be satisfied only approximately at very low frequencies where the radiation impedance becomes small. In such a case, the pole A represents the transfer function Q_1/Q_2, which is the insertion loss of the system, which is driven by an ideal constant flow source. Judging from the pole A in Fig. 9a, the transfer functions estimated by the three different methods will be within 1–2 dB from each other in most of the frequency range except around 770 Hz. This observation indicates that the experimental method provides accurate results enough to be used in practice.

As it can be seen in the comparison, the method provides accurate results in a broad frequency range. Kulkarni et al. reported that the method obtained more accurate results in the low-frequency range unlike the method developed by Doige and his colleagues [1–4], which showed large errors at low frequencies. The accuracy may be attributed to the simplicity of the procedure. The measurement of flow velocities by the two-microphone method is the only part that involves sizable errors in the procedure; therefore the valid frequency

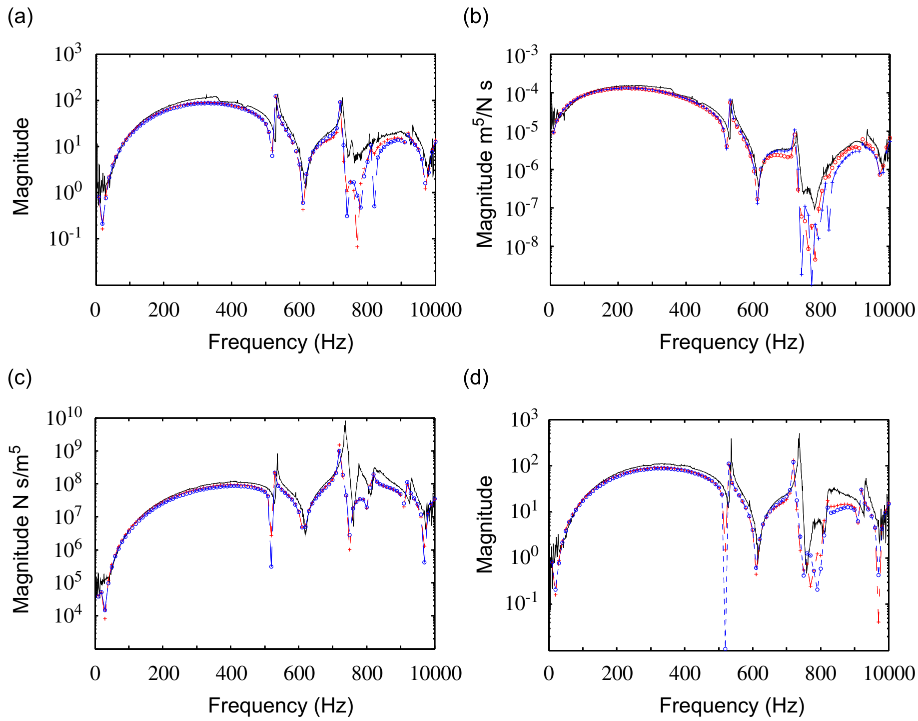

Fig. 9. Four-pole parameters for cavity with short pipes: (a) Pole A, (b) Pole B, (c) Pole C, (d) Pole D, ———: experimental, – –O– –O–·: analytical, –·–+–·–+–·–·: numerical.

range of the procedure is nearly the same as that of the two-microphone method. However, the procedure cannot be applied to systems with high mean flows, which is one obvious drawback.

3.2. Hybrid system modeling: an application example

The cascading property is what makes the four-pole approach truly useful. Using the four-pole method, models of various elements in the system can be obtained by any of the analytical, numerical or experimental methods, and are then integrated to form the system model. For example, in an automotive exhaust system, four poles may be formulated analytically for simple elements such as short pipes or small volumes, numerically for a large cavity that has a large cross-sectional area, thus a small mean flow velocity, which has complex geometry but with well defined boundary conditions such as expansion chambers in the muffler, and experimentally for hard-to-model elements such as a cavity filled with porous material or a catalytic converter section, then combined to obtain the system four pole equation.

As a simple demonstration, we consider a situation that we have to design a small muffler to be attached to the output side of the rectangular cavity we used in this study. As shown in Fig. 10, it is further assumed that the small muffler has a 10% of the volume of the cavity, and we want to compare the three designs shown in Table 1. Again, it is noted that the lengths shown include the end correction effect; therefore are effective lengths. The system model can be made by combining the four poles we measured for the cavity with the analytical four poles of the small volume and pipes. Four poles of a 1-D pipe are given in Eq. (13). The four-pole relationship of a small volume cavity is given by [12]:

$$\begin{Bmatrix} Q_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} 1 & \dfrac{j\omega V_0}{\rho_0 c_0^2} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ P_2 \end{Bmatrix}, \tag{24}$$

where V_0 is the volume of the cavity. The overall four pole of the original system in Fig. 10a can be obtained by multiplying the four pole matrices of the inlet pipe, cavity and the outlet pipe. The four poles of the system

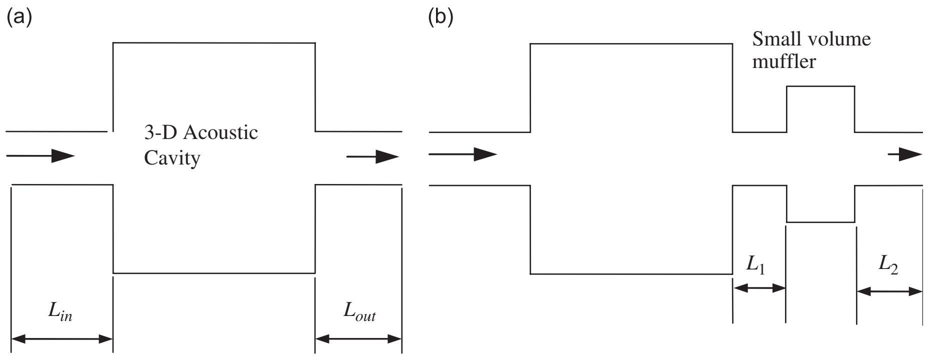

Fig. 10. Expansion chamber with: (a) inlet and outlet pipe and (b) a small volume element in outlet pipe

Table 1
Design by varying the length of outlet pipe

Case	L_1	L_2
1	$0.5L_{\text{out}}$	$0.5L_{\text{out}}$
2	L_{out}	$4L_{\text{out}}$
3	$4L_{\text{out}}$	L_{out}

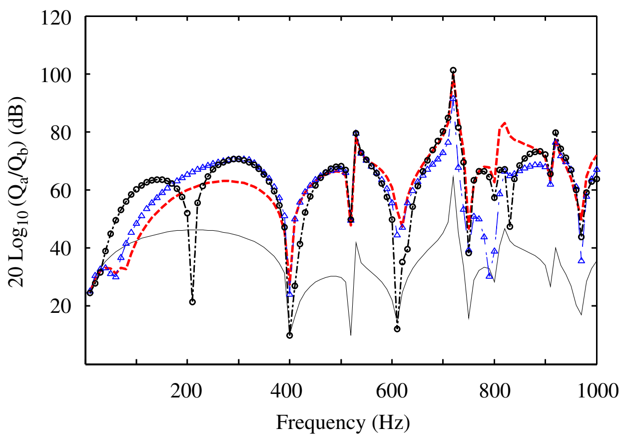

Fig. 11. Transfer function of the system, ———: original system - - - - - - - - -: case 1 (0.5 L_1, 0.5L_2), - -△- - -△- -: case 2 (L_1, 4L_2) —○— · —○— · —: case 3 (4L_1, L_2).

shown in Fig. 10b can be obtained by multiplying 5 four-pole matrices of the pipe L_{in}, the large cavity, the pipe L_1, the small volume and the pipe L_2.

The transfer function to compare the performances of the three designs is defined as follows:

$$\mathrm{TF} = 20\log_{10}\left(\frac{Q_1}{Q_2}\right) = 20\log\frac{1}{A_T}, \tag{25}$$

where Q_1 and Q_2 are input and output volume velocities into and out of the system, respectively, and A_T is the pole A of the overall system. Fig. 11 shows the TF of the three designs compared with the baseline design, the cavity without the muffler shown in Fig. 10a.

Design simulations changing analytical parts of the model, the muffler and 1-D pipes, can be conducted as many times as necessary at virtually no cost. Four poles of the 3-D cavity will have to be obtained only when the new design calls for a different input/output locations and/or a new cavity. Once the procedure is established, experimental formulation of the four poles can be completed in a matter of few hours, which requires an effort necessary comparable to typical numerical analyses. In fact, the approach will take considerably less time than numerical analysis for a very complicated system, not to mention that the result will be more accurate.

4. Conclusions

Since Kim and Soedel [5] proposed it, the procedure to formulate four poles of a general acoustic system in terms of pressure response solutions was applied to 3-D cavities using analytical [7] and numerical methods [6]. In this work, an experimental approach is developed to implement the procedure to measure four poles of a 3-D cavity. The four poles obtained from the experimental procedure are compared with those obtained by analytical and numerical methods to validate the experimental approach for the first time. A rectangular box built with hard wood walls, one of a few 3-D systems that allow all three approaches, is used as the model for this purpose.

In the analytical approach, pressure response solutions of the rectangular cavity, which are necessary to formulate four poles, are obtained by solving the non-homogeneous wave equation using the mode superposition method. It is shown that a very large number of modes are required to obtain a converged pressure response solution at the source point as reported in Ref. [7]. In the numerical approach, pressure response solutions are obtained by using the finite element method. It is shown that the four poles obtained from the numerical and analytical approaches agree very well with each other. In the experimental approach, the pressure response functions are measured using three microphones. The four poles obtained from the experiment agree quite well with those obtained from the analytical and numerical methods, which validates the experimental method.

The main advantage of the experimental method for four-pole formulation is that it can be applied to any acoustic systems. Further, the experimental method developed in this work is so simple that it requires less efforts and time than typical numerical methods if the system becomes very complex. Therefore, the most useful application of the four-pole method will be experimental–analytical–numerical hybrid modeling in that the system equation is formulated by integrating the four poles of sub-systems obtained by, respectively, different method. The approach will be very useful in deign simulations of complex, large-scale acoustic systems. Through a simple application example, such a hybrid modeling concept is demonstrated.

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